Multi-centered Solutions with a (very special) Warped Compactification [1]

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Abstract

We find an exact solution for multi-black strings in the brane world with warped compactification.

1 Introduction

Recently it has been suggested by much work on unification theories that we live in a brane with three-dimensional spatial extension. Randall and Sundrum proposed a warped compactification scenaro [1]. [4] They assumed the five-dimensional spacetime and the presence of a negative cosmological constant in the bulk space. Two branes with tension of individual values realize the compactification of an extra dimension on S^1/Z_2. The mass scale on a brane depends on the warp factor which is determined by the location of the brane in the extra dimension. Thus their model provides a novel solution to the hierarchy problem.

Extensions to many branes have been studied by many authors. (see, for example, [12].) [5]

The models including dilaton fields have been also studied [14] [15]. In the case, although the bulk space is not an anti-de Sitter space, similar warped compactification can be obtained.

On the other hand, the black hole solution on the brane was analyzed by Chamblin et al. [16]. Many other types of localized objects in the brane world may be considered, and we expect that physics on a brane will be clarified by study of them.

In this talk, we show an exact solution for charged objects in the brane world.

2 Brane world

We consider a model including a dilaton field governed by the following action:

$$
\begin{aligned}
S &= \int d^D x \sqrt{-g}\left[R - \frac{4}{D-2}\left(\nabla\phi\right)^2 - 2\Lambda e^{4b\phi/(D-2)}\right] \\
&\quad - \sum_k \int_{brane_k} d^{D-1}x \sqrt{-g_{D-1}} 2\sigma_k e^{2b\phi/(D-2)},
\end{aligned}
\tag{1}
$$

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[4] See also [2, 3, 4, 5, 6, 7, 8, 9, 10, 11] for earlier attempts.
[5] A single brane scenario[13] has been also considered. We will not mention this scenario in detail here.

where Λ is a cosmological constant and σ_k is the tension of the kth brane. The metric is assumed to be

$$ds^2 = \Omega^2(\chi) \left[\eta_{\mu\nu} dx^\mu dx^\nu + d\chi^2 \right], \tag{2}$$

where $\mu, \nu = 0, \cdots, D-2$. We also assume that the configuration of the dilaton field takes the form:

$$e^{4b\phi/(D-2)} = \Omega^{-2b^2}. \tag{3}$$

Under the assumptions, field equations derived from the previous action can be reduced to be

$$(D-2)(D-1-b^2) \left(\frac{\Omega'}{\Omega} \right)^2 = (-2\Lambda)\Omega^{2-2b^2}, \tag{4}$$

$$-\left(\frac{\Omega'}{\Omega} \right)' + (1-b^2) \left(\frac{\Omega'}{\Omega} \right)^2 = \frac{1}{D-2}\Omega^{1-b^2} \sum_k \sigma_k \delta(\chi - \chi_k), \tag{5}$$

where χ_k is a position of the kth brane.

For $b^2 \neq 1$, one can find the following solution:

$$\left(\Omega^{b^2-1} \right)' = \pm(1-b^2)\sqrt{\frac{(-2\Lambda)}{(D-2)(D-1-b^2)}}, \tag{6}$$

$$\left(\Omega^{b^2-1} \right)'' = (1-b^2)\frac{1}{D-2} \sum_k \sigma_k \delta(\chi - \chi_k). \tag{7}$$

This yields the brane solutions on tuning values for brane tensions. [6]

In particular, the case with $b^2 = 0$ corresponds to the Randall-Sundrum model. To see this more explicitly, we introduce a new coodinate $dz \equiv \Omega d\chi$. Then the metric takes the form:

$$ds^2 = e^{-2Kz}\eta_{\mu\nu} dx^\mu dx^\nu + dz^2, \tag{8}$$

where K is a certain constant.

For $b^2 = 1$, the solution is

$$\phi = \pm\frac{\sqrt{-2\Lambda}}{2}\chi, \quad \Omega = e^{-2\phi/(D-2)}. \tag{9}$$

In this case with $b^2 = 1$, we can simplifies the action, if we set $\bar{g}_{\mu\nu} = e^{4\phi/(D-2)}g_{\mu\nu}$, as

$$\begin{aligned} \bar{S} &= \int d^D x \sqrt{-\bar{g}}e^{-2\phi} \left[\bar{R} + 4 \left(\bar{\nabla}\phi \right)^2 - 2\Lambda \right] \\ &- \sum_k \int_{brane_k} d^{D-1}x\sqrt{-\bar{g}_{D-1}}2\sigma_k e^{-2\phi} \end{aligned} \tag{10}$$

Therefore, the bulk solution for $b^2 = 1$ is the linear dilaton solution.

3 Black strings in the brane world

From now on, we concentrate our attention to the $b = 1$ case. We introduce an anti-symmetric tensor field into the model:

$$\begin{aligned} S &= \int d^D x \sqrt{-g} \left[R - \frac{4}{D-2} \left(\nabla\phi \right)^2 - \frac{1}{12}e^{-8\phi/(D-2)}H^2 - 2\Lambda e^{4\phi/(D-2)} \right] \\ &- \sum_k \int_{brane_k} d^{D-1}x\sqrt{-g_{D-1}}2\sigma_k e^{2\phi/(D-2)}, \end{aligned} \tag{11}$$

[6] For a single negative cosmological constant, one should choose $\sigma_k = \pm 2\sqrt{-2\Lambda}$.

where H_{MNL} is an anti-symmetric tensor field strength.

In this model, a solution in the bulk space is:

$$ds^2 = \Omega^2(\chi) \left[h^{-\frac{D-4}{D-2}}(-dt^2 + d\chi^2) + h^{\frac{2}{D-2}}\delta_{ij}dx^i dx^j \right], \tag{12}$$

$$e^{4b\phi/(D-2)} = \Omega^{-2}h^{-\frac{2}{D-2}}, \quad H_{t\chi i} = \partial_i h^{-1}, \tag{13}$$

$$(D-2)^2 \left(\frac{\Omega'}{\Omega} \right)^2 = (-2\Lambda), \tag{14}$$

$$-\left(\frac{\Omega'}{\Omega} \right)' = \frac{1}{D-2}\sum_k \sigma_k \delta(\chi - \chi_k), \tag{15}$$

with

$$h = 1 + \frac{1}{\Omega^{D-2}}\sum_a \frac{\mu_a}{(D-4)|\mathbf{x} - \mathbf{x}_a|^{D-4}}, \tag{16}$$

where μ_a are arbitrary constants.

A spacetime slice with a constant χ appears to be extreme black holes with a dilaton and a scalar field:

$$ds_c^2 = h^{\frac{D-4}{(D-2)(D-3)}} \left[-h^{-\frac{D-4}{D-3}}dt^2 + h^{\frac{1}{D-3}}\delta_{ij}dx^i dx^j \right] + h^{-\frac{D-4}{D-2}}d\chi^2. \tag{17}$$

In particular, for $D=5$, the metric seems

$$ds_c^2 = h^{\frac{1}{6}} \left[-h^{-\frac{1}{2}}dt^2 + h^{\frac{1}{2}}\delta_{ij}dx^i dx^j \right] + h^{-\frac{1}{3}}d\chi^2. \tag{18}$$

4   Induced charge, energy density and pressure

When the black strings penetrate a brane perpendicularly, matter and dilatonic charge distributions are "induced" on the brane. For simplicity, we consider a single brane located at $\chi = 0$. The field equations are:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} - \frac{4}{D-2}\left[\nabla_\mu\phi\nabla_\nu\phi - \frac{1}{2}(\nabla\phi)^2 g_{\mu\nu} \right]$$

$$- \frac{1}{4}e^{-8\phi/(D-2)}\left[H_{\mu\nu}^2 - \frac{1}{6}H^2 g_{\mu\nu} \right] + e^{4\phi/(D-2)}\Lambda g_{\mu\nu} + \frac{\sqrt{-g_{D-1}}}{\sqrt{-g}}e^{2\phi/(D-2)}\sigma g_{\mu\nu}\delta(\chi)$$

$$= \frac{4}{D-2}\frac{\sqrt{-g_{D-1}}}{\sqrt{-g}}e^{2\phi/(D-2)}T_{\mu\nu}, \tag{19}$$

$$- \frac{8}{D-2}\nabla^2\phi - \frac{1}{12}\frac{8}{D-2}e^{-8\phi/(D-2)}H^2$$

$$+ \frac{8}{D-2}e^{4\phi/(D-2)}\Lambda + \frac{4}{D-2}\frac{\sqrt{-g_{D-1}}}{\sqrt{-g}}e^{2\phi/(D-2)}\sigma\delta(\chi)$$

$$= \frac{4}{D-2}\frac{\sqrt{-g_{D-1}}}{\sqrt{-g}}e^{2\phi/(D-2)}Q, \tag{20}$$

where $T_{\mu\nu}$ represents the "induced" energy-momentum tensor and Q represents the "induced" dilatonic charge on the brane, when the bulk solution of black strings in the previous section is substituted into the equations.

We obtain:

$$\rho = -\sigma\left(1 - \frac{1}{\sqrt{h}} \right)\delta(\chi), \tag{21}$$

$$p = -\frac{\sigma}{2}\left(\sqrt{h} + \frac{1}{\sqrt{h}} - 2 \right)\delta(\chi), \tag{22}$$

$$Q = -\rho, \tag{23}$$

where $T^\mu_\nu = diag.(-\rho, p, \cdots, p)$. Here we omitted the singularity of the string core.

In the case with a single black string at the origin in $D = 5$, ρ behaves as $1/r$ and p as $1/r^2$, asymptotically at a large distance r.

5 Summary

We have obtained an exact solution for multi-black strings in the brane world with warped compactification. Since the black strings "induce" matter and charge distributions on the brane, we suppose that the theory of gravity on the brane shoud be an "unusual" one.

In future work, we will examine the geodesic motion around the black strings in the brane world. We will also study black hole solutions with and without charges in the models including dilatonic fields, and in the case with a general number of extra dimensions. It is also interesting to study black holes in the case with thick domain walls.

Furthermore, we should investigate the possible relation to the string/M theory.

Note added: After this talk, we found [17], in which various solitons are studied in the brane world with non-trivial dilatons. The authors would like to thank Prof. K. Akama for information on the earlier work concerning warped compactifications.

References

[1] L. Randall and R. Sundrum, Phys. Rev. Lett. **83** (1999), 3370, hep-ph/9905221.

[2] K. Akama, "Pregeometry" in Lecture Notes in Physics, 176, Gauge Theory and Gravitation, Proceedings of the International Symposium on Gauge Theory and Gravitation, Nara, Japan, August 20-24, 1982, edited by K. Kikkawa, N. Nakanishi and H. Nariai, (Springer-Verlag, 1983), 267-271.

[3] V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. **125B** (1983), 136.

[4] B. Holdom, Stanford preprint ITP-744 (1983); Nucl. Phys. **B233** (1983), 413.

[5] M. D. Maia, J. Math. Phys. **25** (1984), 2090.

[6] C. G. Callan Jr. and J. A. Harvey, Nucl. Phys. **B250** (1985), 427.

[7] M. Visser, Phys. Lett. **B159** (1985), 22.

[8] E. J. Squires, Phys. Lett. **167B** (1986), 286.

[9] J. Hughes, J. Liu and J. Polchinski, Phys. Lett. **B180** (1986), 370.

[10] G. W Gibbons and D .L. Wiltshire, Nucl. Phys. **B287** (1987), 717.

[11] K. Akama, Prog. Theor. Phys. **78** (1987), 184; *ibid.* **79** (1988), 1299; *ibid.* **80** (1988), 935.

[12] H. Hatanaka et al., Prog. Theor. Phys. **102** (1999), 1213, hep-th/9909076.

[13] L. Randall and R. Sundrum, Phys. Rev. Lett. **83** (1999), 4690, hep-th/9906064.

[14] A. Kehagias, hep-th/9906204.

[15] E. Halyo, hep-th/9909127.

[16] J. W. Chamblin, S. Hawking and Reall, hep-th/9909205.

[17] D. Youm, hep-th/9911218.